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Circuit Function Characterizing Tunability of Resonators

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***Abstract*—In this paper we study a sensitivity function of resonance frequencies to capacitance change in resonators with capacitive tuning, which characterizes the local tunability. This function is a circuit function, similar to the input and transmission functions. The features of the sensitivity function, leading to tunability increasing, are defined. The founded features are used to construct resonators from segments of transmission lines with an extended tuning range. The tunability limitations of resonators tunable by one or several capacitors are determined. Synthesized transmission line resonators can be used in tunable bandpass filters. They allow the filter to be tuned over a wider frequency range for a given capacitance change $C_{\mathbf{max}}/C_{\mathbf{min}}$. If the tuning range is specified, the filter will perform the adjustment at a lower ratio $C_{\mathbf{max}}/C_{\mathbf{min}}$. This leads to a smaller change in the insertion loss of the filter in a given frequency range.**

***Index Terms*—Tunable resonator, tenability limitations, circuit function, input susceptance, critical frequencies.**

I. Introduction

RECENTLY, interest in electrically tunable/reconfigurable filters [1]–[6] has increased. Such filters are used in diplexers [5], duplexers [6], and as standalone devices [7], [8].

Most of these filters contain resonators from segments of transmission lines and variable capacitors as elements of tuning. Semiconductor varactors [7]–[11] ferroelectric capacitors [12], [13], sets of lumped capacitors commutated by MEMS switches [14] or *pin* diodes [15] are used as variable capacitors. These resonators have a lot of resonant frequencies ω_n, $n = 1, 2, \ldots$. Of practical interest is a tuning at one resonance frequency ω_n in a determined frequency band $\omega_n \in [\omega_{n\,\min}, \omega_{n\,\max}]$.

The frequency tunability of resonators is ability to change the resonance frequency when capacitance is changed. The tunability of the resonator in a frequency band is characterized by the relationship between frequency ratio $K_n = \omega_{n\max}/\omega_{n\min}$ and the relative variation of the capacitance $\chi = C_{\max}/C_{\min}$. Increasing the tunability of resonators means increasing K_n for specified value of χ or reducing χ for specified value of K_n.

Manuscript received June 5, 2019; revised August 5, 2019 and August 28, 2019; accepted September 2, 2019. This work was supported by the Ukrainian Ministry of Education under Project 0119U100622. This article was recommended by Associate Editor A. Elwakil. *(Corresponding author: Alexander Zakharov.)*

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Digital Object Identifier 10.1109/TCSI.2019.2940066

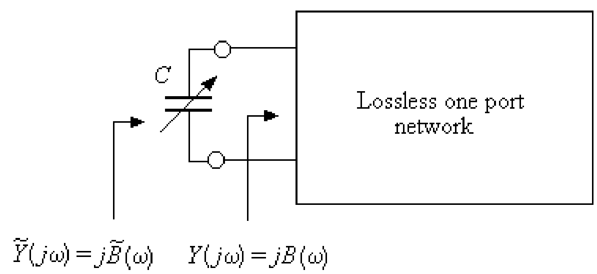

Fig. 1. Tunable resonator with a variable capacitance.

Since the values K_n and χ are not functions of the electrical circuit, the problem of tunability increasing of resonator is solved by enumerating the resonator parameters that improve the relationship between K_n and χ, as was done in [10]. The explanation of the obtained result is not given, as a rule. The existing state is due to the fact that until now there is no circuit function (similar to input or transfer function), which directly expresses the tunability of resonators. We introduce a circuit function characterizing the tunability and study its properties in this paper. For this purpose, we use elements of the sensitivity theory [16]–[19], which has proved itself in many applications. Based on this function we synthesize several versions of resonators with improved tunability and also establish theoretical limitations on tunability of resonators with one and several capacitances.

II. Sensitivity Function and Its Properties

Let us consider a tunable resonator (Fig. 1) consisting of a one-port network wite capacitance C is connected. The one-port network can contain distributed and lumped elements. When the capacitance changes, the resonance frequencies of the resonator ω_n, $n = 1, 2, \ldots,$ change as well. They are the roots of the resonance equation

$$\tilde{B}(\omega) = B(\omega) + \omega C = 0, \tag{1}$$

where $B(\omega)$ is input susceptance of a one-port network, $\tilde{B}(\omega)$ is input susceptance of the resonator.

A positive definite value

$$S_c = -\left.\frac{d\omega/\omega}{dC/C}\right|_{\omega=\omega_n} = -\left.\frac{d\ln\omega}{d\ln C}\right|_{\omega=\omega_n} \tag{2}$$

is the sensitivity of the resonance frequencies ω_n to capacitance C changing. It is defined in the region of negative values

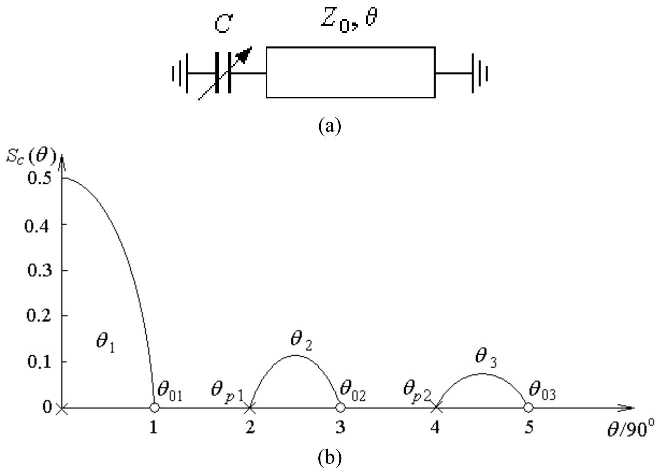

Fig. 2. Tunable combline resonator. (a) Schematic. (b) Function $S_c(\theta)$.

of $B(\omega)$, where (1) has solutions. Using resonance equation (1) we can express the sensitivity S_c (2) through input susceptance $B(\omega)$

$$S_c = \left(1 - \frac{dB}{d\omega} / \frac{B}{\omega}\right)^{-1}\Bigg|_{\omega=\omega_n}, \quad B \leq 0 \tag{3}$$

If function $B(\omega)$ is given by its zeros $\omega_{0\ i}$ and poles ω_{pi}, $i = 1, 2, \ldots$(these frequencies are also called critical)

$$B(\omega) = -H \frac{\left(\omega_{01}^2 - \omega^2\right)\left(\omega_{02}^2 - \omega^2\right)\ldots}{\omega\left(\omega_{p1}^2 - \omega^2\right)\left(\omega_{p2}^2 - \omega^2\right)\ldots}, \tag{4}$$

then the sensitivity is expressed through these critical frequencies

$$S_c(\omega) = \frac{1}{2}\left[1 - \sum_i \frac{1}{1-(\omega_{0i}/\omega)^2} + \sum_i \frac{1}{1-\left(\omega_{pi}/\omega\right)^2}\right]^{-1}, \tag{5}$$

which follows from (3), (4). Here H is a constant value, called conductivity coefficient, which has no effect on sensitivity S_c. The applicability of sensitivity S_c is due to the fact that S_c is circuit function, in contrast to the values K and χ. It contains information about properties of a circuit and it can be determined by changing critical frequencies $\omega_{0\ i}$ and ω_{pi}.

A combline resonator is used most widely in varactor-tuned filters [Fig. 2(a)]. The input susceptance of its line segment

$$B(\omega) = -Z_0^{-1} cot\theta,$$

where Z_0 is the characteristic impedance, $\theta = \omega L/\upsilon$ is the electric length, L is the length of the segment, and υ is the propagation speed of electromagnetic wave. Substitution of this expression in (3) leads to the sensitivity function of this resonator

$$S_c(\theta) = \left(1 + \frac{2\theta}{\sin 2\theta}\right)^{-1}, \quad S_c(\theta) \geq 0. \tag{6}$$

The graph of the sensitivity function (6) for the first three oscillations is shown in Fig. 2(b). The abscissa is the electric length normalized to 90°.

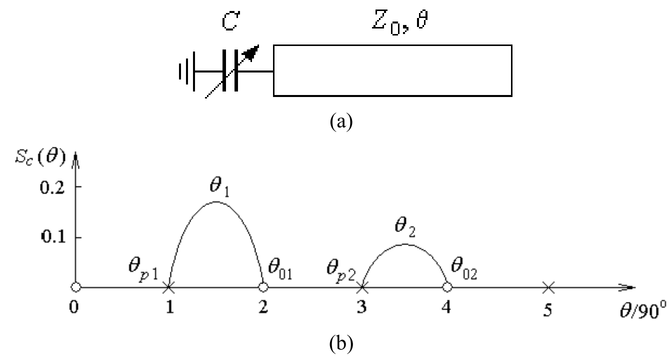

Fig. 3. Tunable resonator with open end. (a) Schematic. (b) Function $S_c(\theta)$.

A resonator shown in Fig. 3(a) also uses as a tunable one. One end of the resonator is open. The input susceptance $B(\omega)$ is expressed by the formula $B(\omega)Z_0^{-1}\tan\theta$. Substitution of this formula into (3) gives the sensitivity function of this resonator

$$S_c(\theta) = \left(1 - \frac{2\theta}{\sin 2\theta}\right)^{-1}, \quad S_c(\theta) \geq 0. \tag{7}$$

The graph of the function S_c (7) for the first two oscillations is shown in Fig. 3(b).

A positive-definite circuit function $S_c(\omega)$ is characterized by the following properties:

– the coincidence of the zeros of the sensitivity function with the internal critical frequencies of the function $B(\omega)$;

– the strict convexity of the function $S_c(\omega)$ between its zeros.

The sensitivity S_c characterizes the local tunability of the resonator. At the same time, it connects the values K and χ. If the sensitivity change in the tuning band from $\omega_{n\min}$ to $\omega_{n\max} = K\omega_{n\min}$ is known, then

$$\ln\chi = \int_{\omega_{n\ \min}}^{K\omega_{n\ \min}} \frac{d\ln\omega}{S_c(\omega)} \tag{8}$$

which follows immediately from definition (2). Expression (8) shows that an increasing S_c in a tuning band leads to decreasing of χ for defined value of K.

It is widely known that tunability of a combline resonator at a main resonance frequency ω_1 increases when an electrical length of the resonator decreases. This fact is explained by the function S_c [Fig. 2(b)]. The sensitivity function increases with decreasing θ up to the value of 1/2. For a resonator with an open end, the function S_c at ω_1 has a maximum [Fig. 3(b)]. For more effective tuning this maximum should be situated in the middle of the tuning range.

It is important to note that the sensitivity function S_c (5) depends only on the critical frequencies. It does not depend on the level of input susceptance H (4) and on capacitance C. This feature is retained when transmission line resonators are used. In this case, the function S_c does not depend on capacitance C and characteristic impedance Z_0. It depends only on the electrical length θ of the transmission line segment (6), (7). Three main values Z_0, C, and θ are used for design of a tunable resonator. Due to the existing feature of S_c

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function, the electrical length θ is determined first. The values of Z_0 and C are determined secondarily. For this, based on the resonance equations of the resonators shown in the Fig. 2(a) and Fig. 3(a), we write down at the upper frequency of the tuning range $\omega_{1\max}$ as $Z_0 C_{\min} = \omega_{1\max}^{-1}\cot(\theta_{1\max})$ and $Z_0 C_{\min} = -\omega_{1\max}^{-1}\tan(\theta_{1\max})$, respectively. The right side of these equalities is a positive number. We denote it by A and write these equations in a single form $Z_0 C_{\min} = A$. This representation shows that the values of Z_0 and $C_{\min}$ are mathematically equal rights. In addition, they are inversely proportional to each other, increasing the value of Z_0 leads to a decrease in the value of $C_{\min}$, and vice versa. The choice of Z_0 and $C_{\min}$ values is somewhat arbitrary. This choice is made by designer, and the main factor in choosing is the equality of the product of Z_0 and $C_{\min}$ values to the number of A. The electrical length θ is determined first by the condition of obtaining the highest possible sensitivity S_c. This is equivalent to minimizing the relative capacitance change $\chi = C_{\max}/C_{\min}$. Therefore, the starting point of the design is the minimization of the value χ.

III. Tunability Limitation of Resonators

The use of the sensitivity function $S_c(\omega)$ allows us to determine the tunability limitation of resonators in general terms. We consider some different cases.

A. Resonator Without Losses With One Capacitance

If the sensitivity is constant [$S_c(\omega) = S_0 = \text{const}$] in the tuning band, then from (8) it follows $\chi = K_n^{1/S_0}$ or $K_n = \chi^{S_0}$. The last equality makes it obvious that the maximum value of the sensitivity function $S_{c\max}$ in the tuning band limits the frequency ratio K_n:

$$K_n \le \chi^{(S_c)_{\max}} \tag{9}$$

The input conductance of a lossless one-port network $B(\omega)$ is a reactance function. The fundamental property of a lossless one-port network is expressed by the inequality [20]

$$\frac{dB}{d\omega} \ge \frac{|B|}{\omega}. \tag{10}$$

Inequality (10) and definition (3) lead to an important property of the function $S_c(\omega)$ of the resonator without losses — limitation of this function on all frequency axis

$$S_c(\omega) \le 1/2. \tag{11}$$

In addition, (11) and (9) lead to limitation of the frequency ratio

$$K_n \le \sqrt{\chi}. \tag{12}$$

Limitations (11) and (12) are fulfilled with an equal sign only in the case when a tunable resonator is a lumped LC circuit.

B. Resonator Without Losses With Several Capacitances

Initially we consider a one-port network (Fig. 1), which contains lumped inductances of constant value and m variable capacitances: $C_1, C_2, \ldots, C_m$. Without loss of generality, we shall consider the lowest resonance frequency ω_1, which depends on these capacitances

$$\omega_1 = \omega_1(C_1, C_2, \ldots, C_m). \tag{13}$$

The function (13) is a homogeneous function of the variables C_i, $i = 1, 2, \ldots, m$, of degree (– 1/2):

$$\omega_1(\chi C_1,\ \chi C_2,\ \ldots,\ \chi C_m) = \chi^{-1/2}\omega_1(C_1,\ C_2, \ldots,\ C_m); \tag{14}$$

$$\sum_{i=1}^{m} C_i \frac{\partial \omega_1}{\partial C_1} = -\frac{1}{2}\omega_1. \tag{15}$$

In particular, we have $\omega_1(C) = \text{const} \cdot C^{-1/2}$ for single LC circuit, where const $= L^{-1/2}$. In this case expressions (14) and (15) become obvious $\omega_1(\chi C) = \chi^{-1/2}\omega_1(C)$ and$C(d\omega_1/dC) == -(1/2)\omega_1$. Definitions (14) and (15) of a homogeneous function of degree (– 1/2) are equal. Equation (14) means that increasing of all capacitances by χ times is accompanied by decreasing of ω_1 by $\sqrt{\chi}$ times. Formula (15) expresses the Euler theorem on homogeneous functions [21]. After dividing both sides of (15) by (– ω_1) we obtain

$$\sum_{i=1}^{m} S_{c_i} = 1/2 \tag{16}$$

where S_{c_i} is sensitivity of resonance frequency ω_1 to change of capacitance C_i(2). Expression (16) is known [16], [17] as invariant of sensitivity ω_1 to the change of all capacitances of a lumped LC circuit. It shows that the sum of all sensitivities S_{c_i} of a lumped circuit without losses is 1/2 and this sum is invariant with respect to the structure of circuit. Equation (16) expresses the mutual dependence of sensitivities, which is one of the fundamental properties of such circuits. The increase in one sensitivity is accompanied by a decrease in other sensitivities, but their sum remains constant.

If several inductances are added to the initial circuit, then the character of the function (13) does not change. It will still be a homogeneous function of degree (– 1/2) and satisfy conditions (14)–(16). However, if several constant capacitances are added to the initial circuit (let their total number become equal to M), then (16) goes to the inequality

$$\sum_{i=1}^{m} S_{c_i} \le 1/2, \tag{17}$$

(since the number of variable capacitances is $m < M$ on which the summation is carried out), then the restriction for the frequency ratio will take the form

$$K \le \sqrt{\chi_{\max}} \tag{18}$$

where $\chi_{\max}$ is the largest value among $\chi_1, \chi_2, \ldots, \chi_m$.

A circuit with distributed parameters without losses can be regarded as the limiting case of a lumped LC circuit, when the number of elements tends to infinity. Adding a

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distributed element (several elements) to initial lumped circuit is equivalent to connect an infinite number of lumped constant capacitances and inductances. Therefore, the limitations of tunability (17), (18) also apply to distributed-lumped circuits without losses, which are tuned by several capacitors. The limitations of tunability of resonators with several variable capacitances (17), (18) and with one variable capacitance (11), (12) coincide. This means that you can not significantly increase the tunability by increasing the number of variable capacitances. This position was confirmed in [9], where the resonator was tuned by three varactors.

In the varactor-tuned bandbass filter [3] microstrip transmission line resonators were used. For tuning in the frequency range of 225-400 MHz ($K = 1.778$), one varactor was used in each resonator, the capacitance of which changed in $\chi = 3.78$ times. The S_c values were quite large $0.402 \leq S_c \leq 0.470$. The use of additional varactors will slightly reduce the value of χ. This suggests that it is not advisable to use additional varactors in the case of high value S_c.

C. Influence of Dissipative Losses

In the presence of losses, the resonant frequencies of resonators will be determined by two conditions

$$\tilde{B}(\omega) = B(\omega) + \omega C = 0$$
$$\left.\frac{d\tilde{B}}{d\omega}\right|_{\omega=\omega_n} = C + \left.\frac{dB}{d\omega}\right|_{\omega=\omega_n} \geq 0 \qquad (19)$$

where functions $\tilde{B}(\omega)$ and $B(\omega)$ are no longer reactance functions. They are imaginary parts of the corresponding input admittance $\tilde{Y}(\omega)$ and $Y(\omega)$. The first of conditions (19) coincides with the resonance condition for the case without loss (1), but its solution gives a large number of zeros on frequency axis. Some of these zeros are not resonant frequencies, since the function $\tilde{B}(\omega)$ at these frequencies has a negative slope. The second of conditions (19) chooses those zeros where function $\tilde{B}(\omega)$ has a positive slope and they are resonant frequencies. When losses are absent, the second of conditions (19) is redundant, since the slope of the input susceptance is always positive.

Since the first of resonance conditions (19) gives $C = -B/\omega$ the second of conditions (19) is equivalent to the inequality

$$\frac{dB}{d\omega} \geq \left.\frac{|B|}{\omega}\right|_{\omega=\omega_n}.$$

This inequality coincides with inequality (10) for the input susceptance, which led to the limitation of tunability (11), (12). The same inequalities hold also for resonators with losses. Therefore, the idea to increase the tunability of resonators due to the artificial adding of losses in them has no real basis.

As a rule, the operating frequencies of a resonator with fixed tuning are located in the frequency band with a relative attenuation level of -3 dB. The smaller the unloaded Q_u of the resonator, the wider this band. In view of above, if the resonator is tuning in the frequency range from $\omega_{\min}$ to $\omega_{\max}$, then the operating frequency range should be considered extended. However, this effect is not significant.

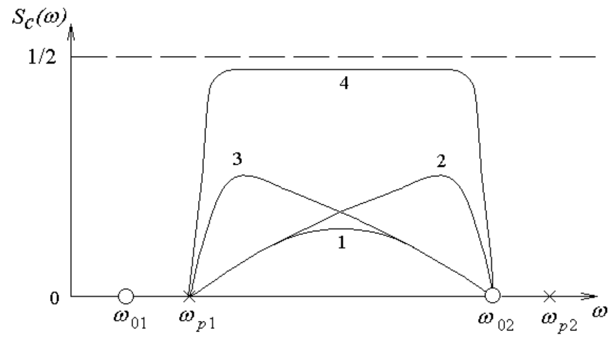

Fig. 4. Dependence of function $S_c(\omega)$ on the location of critical frequencies: 1) $\omega_{01} \to 0$, $\omega_{p2} \to \infty$; 2) $\omega_{01} \to \omega_{p1}$, $\omega_{p2} \to \infty$; 3) $\omega_{p2} \to \omega_{02}$, $\omega_{01} \to 0$; 4) $\omega_{01} \to \omega_{p1}$, $\omega_{p2} \to \omega_{02}$.

IV. Ways of S_c Increasing

Since $S_c(\omega)$ is a circuit function a natural question arises. How to increase $S_c(\omega)$ due to a special distribution of critical frequencies $\omega_{0\ i}$, ω_{pi}?

On the example of the function $S_c(\omega)$ determined by critical frequencies ω_{01}, ω_{p1}, ω_{02}, ω_{p2} (Fig. 4), we consider the possibility of increasing the sensitivity at the second resonant frequency $\omega_2 \in [\omega_{p1}, \omega_{02}]$. Obviously, that increasing of frequency interval $[\omega_{p1}, \omega_{02}]$, i.e. expansion of the resonance region, will lead to increasing in the values of $S_c(\omega)$. Fig. 4 illustrates the effect of critical frequencies ω_{01} and ω_{p2} located to the left and right of the resonance region. In the case under consideration, the sensitivity function (5) contains four critical frequencies ω_{01}, ω_{p1}, ω_{02}, ω_{p2}.

Curve 1 in Fig. 4 corresponds to the location of the critical frequencies $\omega_{01} \to 0$, $\omega_{p2} \to \infty$. In this case the sensitivity takes the smallest values. Curve 2 was obtained at $\omega_{p2} \to \omega_{02}$, $\omega_{01} \to 0$ and curve 3 corresponds to $\omega_{01} \to \omega_{p1}$, $\omega_{p2} \to \infty$. Oncoming to frequency interval $[\omega_{p1}, \omega_{02}]$ of only one critical frequency increases $S_c(\omega)$ in the region shifted towards this frequency. If two critical frequencies ω_{01} and ω_{p2} are closing to interval $[\omega_{p1}, \omega_{02}]$ simultaneously, then the sensitivity $S_c(\omega)$ in this interval will increase substantially (curve 4 in Fig. 4).

As we can see above, the most wideband resonator is a lumped LC circuit. In this case the limitations (11) and (12) are fulfilled with an equal sign. Then the input susceptance (4) is the inductance conductivity

$$B(\omega) = -\frac{H}{\omega}, \qquad (20')$$

where $H = 1/L$. According to (2), this inductance conductivity corresponds to the sensitivity function equal to 1/2 on the entire frequency axis

$$S_c(\omega) = 1/2. \qquad (20'')$$

The function (20′) has no internal critical frequencies. It has a pole at zero frequency and zero at infinity.

The question arises could we build a circuit whose input susceptance $B(\omega)$ contains internal critical frequencies (4) with sensitivity (5) close to 1/2 on the entire frequency axis. Using the analysis results presented in Fig. 4 we construct the function $B(\omega)$ as follows. Let all internal poles ω_{pi} of the function differ from zeros $\omega_{0\ i}$ by a small positive value ε: $\omega_{pi} - \omega_{0\ i} = \varepsilon$. The function has a pole at zero frequency and a zero at infinity. An example of such a function is shown in Fig. 5(a), where the dotted line shows the inductance conductivity (20′). At $\varepsilon \to 0$ we obtain the idealized function $B^*(\omega)$, whose module can be represented as

$$\left|B^*(\omega)\right| = \frac{H}{\omega} + \sum_i \delta\left(\omega - \omega_{pi}\right), \qquad (21')$$

where $\delta(\omega - \omega_{pi})$ is the delta-Dirac function [21]. The idealized function $B^*(\omega)$ differs from the inductance conductivity (20′) by the presence of discontinuities of the second kind, where it takes the values $\pm\infty$. The idealized input susceptance (21′) corresponds to the idealized sensitivity function

$$S_c^*(\omega) = \begin{cases} 1/2 & \text{at } \omega \neq \omega_{pi} \\ 0 & \text{at } \omega = \omega_{pi}. \end{cases} \qquad (21'')$$

The function (21″) is shown in Fig. 5(b). The input reactance $X(\omega) = -1/B(\omega)$ corresponds to the input susceptance $B(\omega)$ with pairwise close critical frequencies. The zeros $\omega'_{0\ i}$ and poles ω'_{pi} of this function are also pairwise close. An example of a such function is presented in Fig. 5(c), where the dotted line shows the inductance reactance $X(\omega) == L\omega$. Extremely closing the critical frequencies $\omega'_{0\ i}$ and ω'_{pi} to each other we obtain an idealized input reactance $X^*(\omega)$, whose modulus can be represented as

$$\left|X^*(\omega)\right| = H\omega + \sum_i \delta\left(\omega - \omega'_{pi}\right).$$

This idealized function differs from the reactance of inductance by the presence of discontinuities of the second kind, where it takes the values of $\pm\infty$. The graphs in Fig. 5(a) and Fig. 5(c) illustrate the input susceptance and input reactance with pairwise close critical frequencies.

Let us give a qualitative interpretation of the result presented in Fig. 5. Expression (5) shows that the pairwise convergence of the zeros and the adjacent right-hand poles of the function $B(\omega)$ bring mutual compensation of their influence on the sensitivity function $S(\omega)$ of the circuit. The influence of the pole and zero located at the origin and at infinity remains uncompensated. As a result, a sensitivity close to inductance is provided. This way of tunability increasing can be called the method of mutual compensation of zeroes and poles of input susceptance.

We illustrate this method by the example of a quarter-wave resonator with a length L and characteristic impedance Z_0 [Fig. 6(a)]. The variable capacitance C is connected to the point that is located at the distance l from the short-circuited end of the resonator. At the connection point of the capacitance the input susceptance is determined by the expression:

$$B(\omega) = Z_0^{-1}\{-cot(\omega l/\upsilon) + tan[\omega(L - l)/\upsilon]\} \qquad (22)$$

where υ is the electromagnetic wave propagation velocity. The graph of function (22) is shown in Fig. 6(b) at $l = L/10$. The smaller distance l, the closer zeros $\omega_{0\ i}$ and poles ω_{pi} are to each other. The tunability of resonator in Fig. 6(a) is

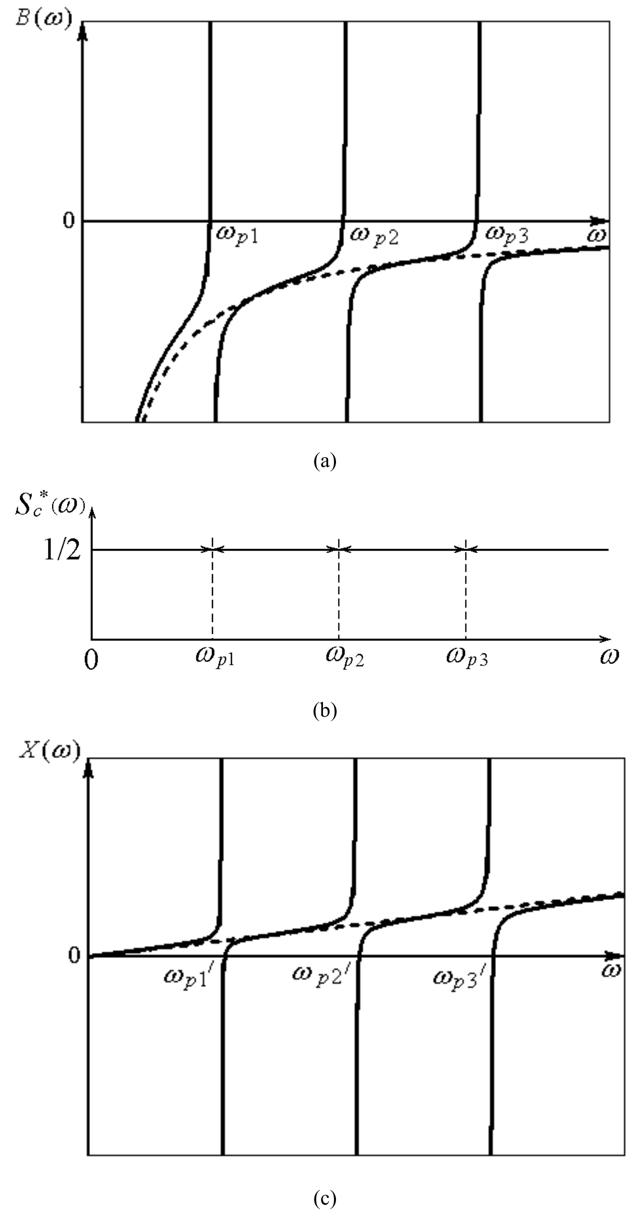

Fig. 5. Characteristic graphs of circuit functions: (a) Input susceptance $B(\omega)$ with pairwise close critical frequencies. (b) Idealized sensitivity function $S_c^*(\omega)$. (c) Input reactance $c(\omega)$ with pairwise close critical frequencies.

higher than that of a resonator with a traditional capacitance connection [Fig. 2(a)], which is tuning in the region of electrical lengths $\theta = 60°$ - $75°$ when capacitance C changes 7.61 times. Using calculations based on (22), it was established that the resonator [Fig. 6(a)] at $l = L/10$ is tuning in the region of the same electrical lengths when the capacitance C changes 4.13 times. However, this significantly increases the minimum value of the capacitance $C_{\min}$. This increasing may be useful at very high frequencies, where the minimum value of capacitance C may be extremely small, for example, $C_{\min} = 0.1$ pF. Moving the connection point of capacitance C towards the short-circuited end of the resonator increases the value of $C_{\min}$.

Each resonant frequency ω_i can change only within its own resonant region, which is limited by zero $\omega_{0\ i}$ and left adjacent

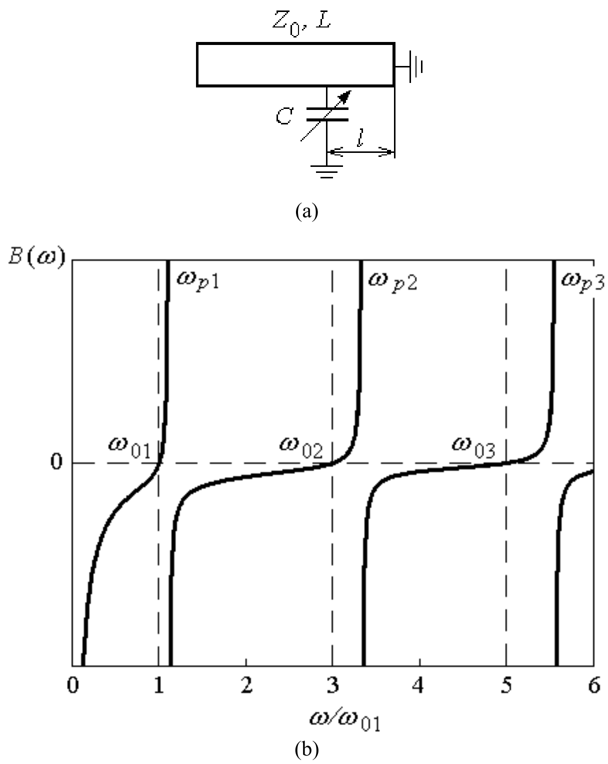

Fig. 6. Implementation example of the mutual compensation method of zeros and poles of input susceptance: (a) Resonator circuit. (b) Input susceptance at the point of connecting the capacitor to the resonator for $l = L/10$.

pole $\omega_{p(i-1)}$ of input susceptance $B(\omega)$. The expansion of the resonance region is accompanied by an increase in sensitivity S_c and frequency range at the corresponding resonant frequency.

When design tunable resonators it is necessary to use both ways of tunability increasing. It is expansion of resonance region and using of the method of mutual compensation of zeros and poles of the input susceptance $B(\omega)$.

V. Resonators With Improved Tunability

The above features of the critical frequencies location of the functions $S_c(\omega)$ and the input susceptance $Y(j\omega) = jB(\omega)$ were used to construct resonators with improved tunability. These resonators are presented in Table 1. They are stepped-impedance resonators with electric length θ consisting of N segments of the same length $\theta' = \theta/N$. The characteristic impedances of composite segments have only two values, the minimum Z_0 and the maximum mZ_0, $m > 1$. The arrow shows the input of each resonator to which the variable capacitance is connected. Resonators No. 1 and No. 2 are designed for tuning the first (fundamental) resonant frequency ω_1. With the help of resonators No. 3 and No. 4 the second resonance frequency ω_2 is tuning. Table 1 also shows the location of the critical frequencies of these resonators, providing improved tunability. They include both the expansion of a resonance region and closing of neighboring critical frequencies to it.

At the Richards frequency variable [22]

$$S = j\Omega = j\tan\theta' \tag{23}$$

the transfer matrix of stepped-impedance resonators is written in the form [23]

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \left(1 - S^2\right)^{-N/2} \times \begin{vmatrix} a_0 + a_2S^2 + \ldots + a_NS^N & a_1S + a_3S^3 + \ldots + a_{N-1}S^{N-1} \\ b_1S + b_3S^3 + \ldots + b_{N-1}S^{N-1} & b_0 + b_2S^2 + \ldots + b_NS^N \end{vmatrix} \tag{24}$$

where N is even number. When N is odd, the higher powers of the polynomials are interchanged, $a_0 = b_0 = 1$. The remaining coefficients of the polynomials are positive numbers that depend on characteristic impedances $Z_{0\,i}$, $i = 1, 2, \ldots, N$. The input susceptance of N-stage resonator is described by the reactance function of variable S (23) of degree N. If a resonator is open at the ends, then from (24) we get

$$Y^N(S) = \frac{C}{A} = \frac{a_0 + a_2S^2 + \ldots + a_NS^{N-1}}{b_1S + b_3S^3 + \ldots + b_{N-1}S^N} \tag{25a}$$

$$= H\frac{S\left(S^2+\Omega_{01}^2\right)\left(S^2+\Omega_{02}^2\right)\ldots\left(S^2+\Omega_{0\,(N-2)/2}^2\right)}{\left(S^2+\Omega_{p1}^2\right)\left(S^2+\Omega_{p2}^2\right)\ldots\left(S^2+\Omega_{p\,N/2}^2\right)} \tag{25b}$$

where H is the conductivity coefficient, $\Omega_{0\,j}$ and Ω_{pj} are critical frequencies at the axis $j\Omega$. If a resonator is short at the end, then $Y^N(S) = D/B$ and even and odd polynomials in (25a), (25b) are interchanged. Table II shows the input susceptance of the resonators from Table I and the ratios of their critical frequencies as a function of m. They can be useful in the calculation and evaluation of the achieved effect. The value ω_{01}^{o} of resonator No. 1 is the zero of input susceptance when a resonator is uniform ($m = 1$). At this frequency we have $\theta_{01}^{\circ} == 90°$. At $m = 3$ the ratio $\omega_{01}/\omega_{01}^{\circ} = 1.33$, which indicates an extension of the resonant region by 1.33 times. Its right boundary $\theta_{01}^{\circ} = 90°$ is increased up to $\theta_{01} = 120°$. At $m = 5$ the ratio $\omega_{01}/\omega_{01}^{\circ} = 1.46$, which indicates an increasing of the right boundary up to $\theta_{01} = 131.8°$. The resonator No. 2 at $m == 1$ is a half-wave resonator with critical frequency ratio $\omega_{01}/\omega_{p1} = 2$. At $m = 3$ we have $\omega_{01}/\omega_{p1} = 3.37$ and for $m = 5$ we obtain $\omega_{01}/\omega_{p1} = 4.36$, which indicates a significant expansion of the resonance region.

We note that the expansion effect of the resonant region of resonator No. 1 is maximal when the electrical lengths of two segments with high and low characteristic impedance are equal. If this ratio is changed, the resonance region becomes narrower. For resonator No. 2 the maximum expansion effect of the resonant region occurs when the ratio of the electrical lengths of composite segments is equal to the value that shown in Table 1.

The substitution of the input susceptance of resonator No. 1 (Table 1) into (3) leads to its sensitivity function

$$S_c = \left[1 + \theta\left(1 + \Omega^2\right)\left(\frac{1}{\Omega} + \frac{2\Omega}{m - \Omega^2}\right)\right]^{-1} \tag{26}$$

where $\Omega = \tan(\theta/2)$. The function (26) is shown in Fig. 7 for $m = 3$ and $m = 1$ (uniform resonator). For $m = 3$ the function S_c increases due to expansion of the resonance

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TABLE I
TUNABLE RESONATORS AND ITS CRITICAL FREQUENCIES

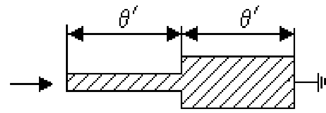
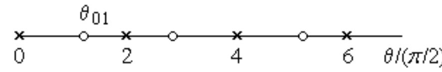
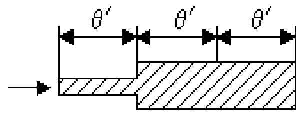
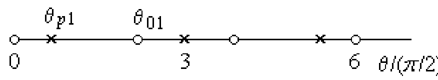
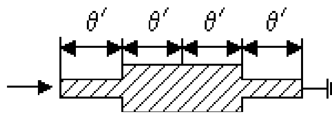
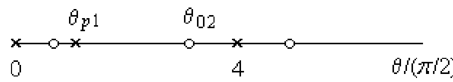
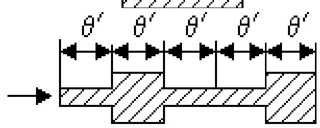
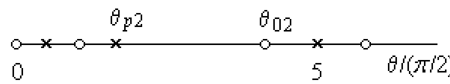
No	Topology of Resonators	Critical Frequencies
1 θ_1		
2 θ_1		
3 θ_2		
4 θ_2		

TABLE II
INPUT SUSCEPTANCE AND RATIO OF THE CRITICAL FREQUENCIES OF TUNABLE RESONATORS

No	Input Susceptance	Ratio of Critical Frequencies
1 ω_1	$Y^2(S)=Z_0^{-1}\dfrac{1+m^{-1}S^2}{(1+1/m)S}$	$\dfrac{\omega_{01}}{\omega_{01}{}^0}=\dfrac{4\tan^{-1}\sqrt{m}}{\pi}$
2 ω_1	$Y^3(S)=Z_0^{-1}\dfrac{(1+2m)S+S^3}{1+(1+2m)S^2}$	$\dfrac{\omega_{01}}{\omega_{p1}}=\dfrac{\tan^{-1}\sqrt{1+2m}}{\tan^{-1}\sqrt{1/(1+2m)}}$
3 ω_2	$Y^4(S)=Z_0^{-1}\dfrac{1+(2+2m+2/m)S^2+S^4}{(2+2/m)S+(2+2m)S^3}$	$\dfrac{\omega_{02}}{\omega_{p1}}=\dfrac{\tan^{-1}\sqrt{1+m+1/m+\sqrt{(1+m+1/m)^2-1}}}{\tan^{-1}\sqrt{1/m}}$
4 ω_2	$Y^5(S)=Z_0^{-1}\dfrac{b_1S+b_3S^3+b_5S^5}{a_0+a_2S^2+a_4S^4}$ $a_0=1,\quad a_2=4+4m+2/m,\quad a_4=1+2m+2m^2$ $b_1=3+2m,\quad b_3=2+4m+2m^2+2/m,\quad b_5=1$	$\dfrac{\omega_{02}}{\omega_{p2}}=\dfrac{\tan^{-1}\sqrt{b_3/2b_5+\sqrt{(b_3/2b_5)^2-b_1/b_5}}}{\tan^{-1}\sqrt{a_2/2a_4+\sqrt{(a_2/2a_4)^2-a_0/a_4}}}$

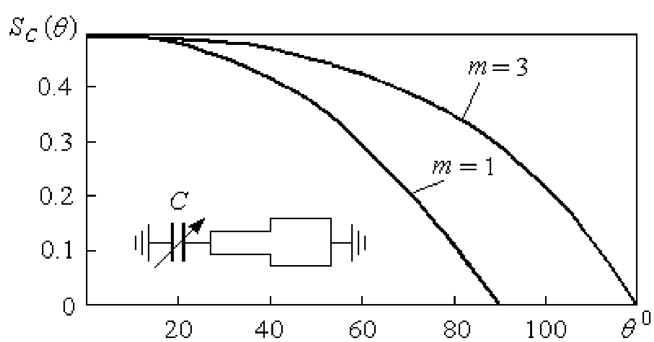


Fig. 7. Function $S_C(\theta)$ of resonator No. 1.

region. It becomes equal to zero for $\theta_{01} = 120°$. The graphs in Fig. 7 show that advantage of stepped-impedance resonator No. 1 increases when tuning in the region with increased electrical lengths. Let us illustrate this for the typical value $\chi = C_{\max}/C_{\min} = 2.2$ for ferroelectric capacitors.

To characterize the frequency adjustment, in practice, the fractional tuning range (FTR) is often used, expressed in percent:

$$\begin{aligned} FTR &= 2(\theta_{n\max} - -\theta_{n\min})/(\theta_{n\max} + \theta_{n\min}) \\ &= 2(K_n - -1)/(K_n + 1). \end{aligned}$$

The FTR values for different electrical lengths of resonator No. 1 are shown in Table III for $m = 3$ and $m = 1$. These values are determined directly from the resonance equation (1). The quantitative evaluation of this advantage is given in Table III as FTR ratio.

The substitution of input susceptance of resonator No. 2 (Table 1) into (3) gives its sensitivity function

$$S_c = \left[1 - \theta\left(1+\Omega^2\right)\left(\frac{2a\Omega}{1-a\Omega^2} + \frac{3\Omega^2 - a}{\Omega^3 - a}\right)\right]^{-1} \quad (27)$$

where $\Omega = \tan(\theta/3)$, $a = 2\ m + 1$.

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TABLE III
TUNING BANDWIDTH OF RESONATOR NO. 1 AT $\chi = 2.2$ FOR $m = 3$ AND $m = 1$

θ (m = 1), degree	40…54	50…64	60…72.4	70…79.4	80…85.1	-
FTR (m = 1), %	29.8	24.6	18.8	12.6	6.2	-
θ (m = 3), degree	40…56.4	50…68.5	60…79.4	70…89.2	80…97.2	90…104.2
FTR (m = 3), %	34	31.2	27.9	24.1	19.4	14.6
FTR (m = 3) / FTR (m = 1)	1.14	1.27	1.48	1.91	3.13	-

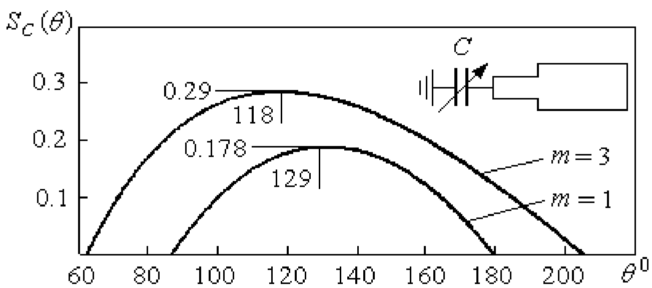

Fig. 8. Function $S_C(\theta)$ of resonator No. 2.

The function (27) is shown in Fig. 8 for $m = 3$ and $m = 1$ and it has a bell-shaped character. In view of this, the resonator has an optimal tuning band, within which the tuning is most effective. At $m = 1$ the maximum of the function $S_{\mathrm{cmax}} = 0.178$ is achieved for electric length $\theta = 129°$, which is located in the middle of this band. Using resonance equation (1) we establish that for $\chi = 2.2$ this band is located in the region of electric lengths of $120° \leq \theta \leq 137.9°$ and its FTR == 13.86%. At $m = 3$ the function S_c increases due to expansion of the resonance region and closing of neighboring critical frequencies to it. The maximum of the function $S_{\mathrm{cmax}} = 0.29$ is achieved for the electric length $\theta = 118°$. At $\chi = 2.2$ the most effective tuning takes place in the region of electric lengths $110° \leq \theta \leq 137.7°$, that corresponds to FTR = 22.36%.

The tuning range is increased 1.61 times in comparison with a nonuniform resonator, for which $m = 1$. As the parameter m increases, the sensitivity function S_c and tuning range of considered stepped-impedance resonators increase as well.

Quarter-wave stepped-impedance resonator (No. 1 in Table I) is not only one that has an increased bandwidth at the main resonant frequency ω_1. Fig. 9 shows resonators, which are equivalent at the frequency ω_1 for the certain capacitances. Their equivalence is due to the fact that at the frequency ω_1 their resonance equations coincide. Taken in Fig. 9 notation requires no explanation. Note that at the frequency of the next resonance ω_2 these resonators are not equivalent, since their resonance equations differ from each other. Practical use given in Fig. 9 resonators is determined by their individual properties. Thus, the resonator [Fig. 9(d)] uses twice value $2C$ of capacitance. In addition, this resonator has an increased length, which is a positive factor at high frequencies.

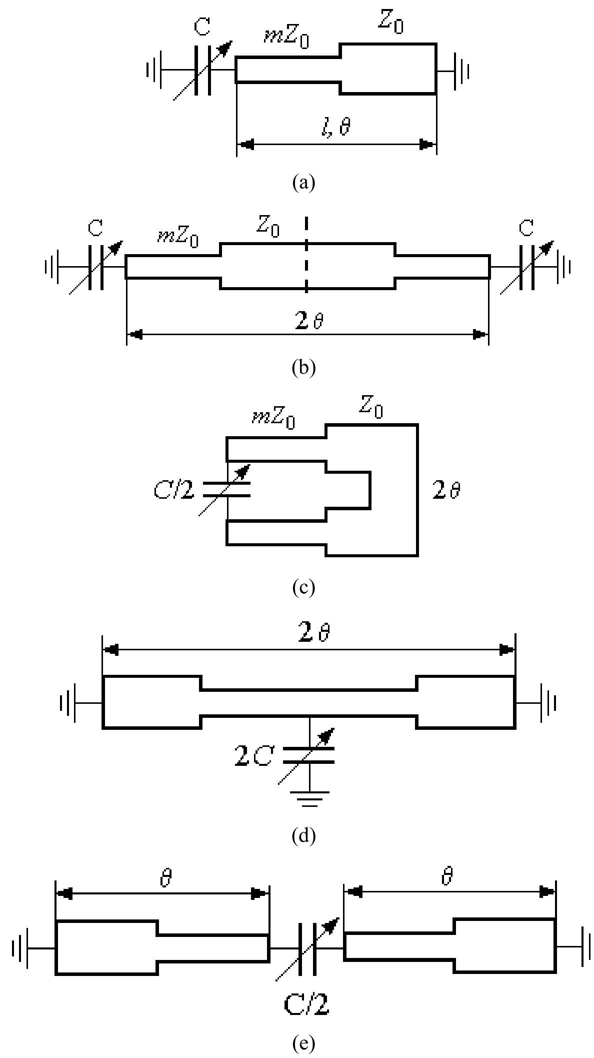

Fig. 9. Resonators with the same tuning range at the main resonant frequency as the quarter-wave SIR: (a) Initial resonator. (b) Symmetric resonator with two capacitors. (c) Loop hairpin resonator. (d) Symmetric resonator with parallel capacitor. (e) Symmetric resonator with series capacitor.

Half-wave stepped-impedance resonator (No. 2 in Table I) is also not only one with increased bandwidth at the main resonant frequency ω_1. Fig. 10 shows resonators, which are equivalent at the frequency ω_1 for the certain capacitances. Taken in Fig. 10 notation requires no explanation. At the same

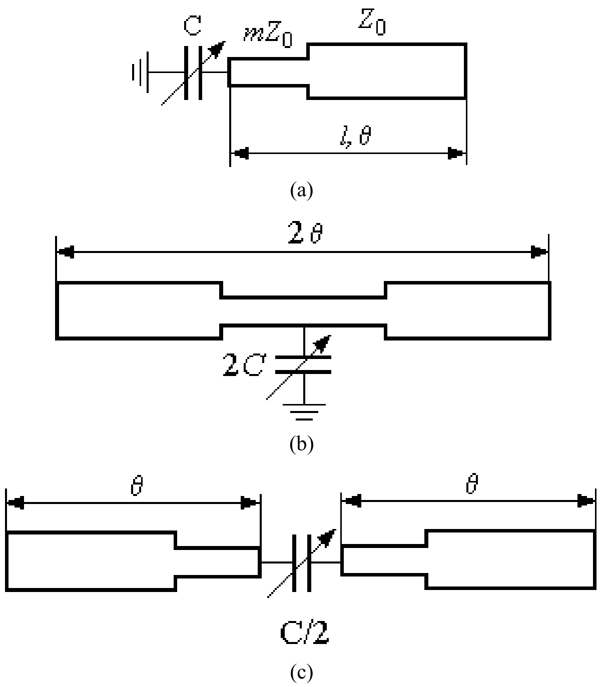

Fig. 10. Resonators with the same range tuning at the main resonant frequency as the half-wave SIR: (a) Initial resonator. (b) Symmetric resonator with parallel capacitor. (c) Symmetric resonator with series capacitor.

time, these resonators are not equivalent at the next resonant frequency ω_2.

The tunable resonator similar to No. 1 was obtained in [10] by EM simulation and step-by-step changing of the ratio between composite lengths of the stepped-impedance resonator. The reason of the tunability increasing of this resonator was not discussed. In work [11] stepped-impedance resonators, shown in Fig. 10(b) and Fig. 10(c), were used. The constructing process of this resonator was not considered. The data [10], [11] indicate the reliability of the results presented in this article.

The resonators No. 3 and No. 4 (Table I and Table II) use the second resonant frequency ω_2. This allows us to use the higher electrical lengths of resonators, which is useful for shorter wavelengths, namely, millimeter. An analysis of these resonators can be carried out in the sequence discussed above.

Table 1 and Fig. 9, Fig. 10 show stepped-impedance resonators with improved tunability. They implement the established features of the critical frequencies location of function $B(\omega)$. An important design parameter of these resonators is $m = Z_{0\max}/Z_{0\min}$. For stripline and microstrip structures the parameter m is limited to the value of $m \approx 5$. In our opinion, the use of distributed stubs circuits [24] is very promising for realization of special arrangement of critical frequencies.

VI. Regularities of Tunable Resonators

Note general patterns of tunable resonators, which become apparent when using the sensitivity function S_c (3), (5).

1. Tunable at main resonant frequency resonators with input susceptance B (4), having a pole at zero frequency, have higher frequency range than resonators with input susceptance having zero at zero frequency. This is explained by the differences in their sensitivity functions $S_c(\omega)$, which are shown in Fig. 2 and Fig. 3, respectively.

2. When the electrical length of resonators with input susceptance $B(\omega)$, having a pole at zero frequency [Fig. 2(b)], decreases, the sensitivity S_c approaches 1/2, and their frequency range tends to the LC circuit. Frequency range comparison of such resonators correctly carried out with the same electrical lengths.

3. The sensitivity function $S_c(\omega)$ of resonators with input susceptance $B(\omega)$, which has zero at zero frequency, has a bell-shaped character at the main resonant frequency [Fig. 3(b)]. Such resonators are characterized by an "optimal" tuning band, in the middle of which the maximum of function $S_c(\omega)$ is located. Frequency range comparison of such resonators can be performed with different electrical lengths from each other.

4. If a resonator with a single variable capacitance has not enough high sensitivity S_c and tunability, then they can be improved by using an additional variable capacitance. For example, a resonator with single variable capacitance (Fig. 8) has a maximum sensitivity $S_{c\max} = 0.29$. Using the second variable capacitance and the transition to resonator [Fig. 9(b)] increases the maximum sensitivity value to 1/2 and improves tunability.

5. If the sensitivity $S_c(\omega)$ of a resonator with single variable capacitance is high enough [3], then the use of additional variable capacitances can not lead to any significant increase in tunability. This is indicated by inequalities (17), (18).

6. The adding of dissipative losses into resonator is not a means of increasing its frequency range, since the restrictions (11), (12) are valid in this case as well.

VII. Conclusion

The introduced sensitivity function $S_c(\omega)$ is a circuit function with the same critical frequencies as the input susceptance. It allows us to determine the location of the critical frequencies, which increase the tunability of resonators with variable capacitance. These features are realized in the class of a distributed circuit, which resulted in the resonators No. 1 — No. 4 in Table I, as well as to the resonators shown in Fig. 9 and Fig. 10. The use of $S_c(\omega)$ function made it possible to formulate recommendations for design and comparison of tunable resonators. Since the function $S_c(\omega)$ is a circuit function, it was used to established the tunability limitation for different cases. We found that in many cases the use of additional variable capacitances cannot lead to a significant improvement in the resonator tunability.

Synthesized transmission line resonators are stepped-impedance resonators. Possibilities to implement different functions of sensitivity $S_c(\omega)$ using distributed circuits of this type are limited by the design parameter $m = Z_{0\max}/Z_{0\min}$. Distributed circuits of a stubs structure could have a wider range of possibilities when implementing various $S_c(\omega)$.

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